# METHOD OF DIFFERENTIATION

## EXERCISE – I HINTS & SOLUTIONS

**Sol.1 C**

$y = f(x)$

$f(-x) = -f(x) \quad \Rightarrow \quad -f'(-x) = -f'(x)$

$f'(3) = f'(-3) = -2$

**Sol.2 A**

$f(x) = \log_x(\ell nx)$, $f'(x)|_{x=e} = ?$

$$f(x) = \log_x(\ell nx) = \frac{\log(\ell nx)}{\log x}$$

$$f'(x) = \frac{\log x \times \frac{1}{x\ell nx} - \frac{1}{x}\log \ell nx}{(\log x)^2}$$

$$f'(x)|_{x=\ell} = \frac{1}{e}$$

**Sol.3 B**

$$y = \cos^{-1}(\cos x), \left.\frac{dy}{dx}\right|_{x=\frac{5\pi}{4}}$$

$$y' = \frac{-1}{\sqrt{1-\cos^2 x}} \times -\sin x = \frac{\sin x}{|\sin x|}$$

$$y'|_{x=\frac{5\pi}{4}} = -1$$

**Sol.4 C**

$$x = \frac{1+t}{t^3}, \ y = \frac{3}{2t^2} + \frac{2}{t}$$

$$\frac{dx}{dt} = -3t^{-4} - 2t^{-3}$$

$$\frac{dy}{dt} = -3t^{-3} - 2t^{-2}$$

$$\frac{dy}{dx} = \frac{-t^2(3t^{-1}+2)}{-t^{-2}(3t^{-1}+2)}$$

$$\left(\frac{dy}{dx}\right) = t\left(\frac{3+2t}{3+2t}\right) = t$$

$x \,.\, t^3 - t = 0$

$$\frac{1+t}{t^3} \times t^3 - t = 1$$

**Sol.5 B**

$\sin(xy) + \cos(xy) = 0$

$\cos xy\,(xy' + y) - \sin(xy)\,(xy' + y) = 0$

$(xy' + y)(\cos xy - \sin xy) = 0$

since $\cos xy \neq \sin xy$, hence

$$y' = -\frac{y}{x}$$

**Sol.6 C**

$y = x^{x^2}$

$n = x^2, \ y = x^n$

$\ell ny = x^2 \ \ell nx$

$y' = x^{x^2}\{x + 2x\ell nx\}$

$y' = x^{x^2+1}(1 + 2\ell nx)$

**Sol.7 D**

$f(x) = |x|^{|\sin x|}$

$x > 0, \ f(x) = x^{\sin x} \Rightarrow y = x^{\sin x}$

$\ell ny = \sin x \ \ell nx$

$$y' = x^{\sin x}\left\{\frac{\sin x}{x} + \cos x \ell nx\right\}$$

$$y|_{x=\frac{\pi}{2}} = \left(\frac{\pi}{4}\right)^{\frac{1}{\sqrt{2}}}\left\{\frac{1}{\sqrt{2}} \times \frac{4}{\pi} + \frac{1}{\sqrt{2}}\ell n\frac{\pi}{4}\right\}$$

$x < 0$

$y = -x^{-\sin x}$

$\ell ny = -\sin x \ \ell n(-x)$

But this case is impossible.

**Sol.8 C**

$$y = \sin^{-1}\left(\frac{x^2-1}{x^2+1}\right) + \sec^{-1}\left(\frac{x^2+1}{x^2-1}\right)$$

$$y = \operatorname{cosec}^{-1}\left(\frac{x^2+1}{x^2-1}\right) + \sec^{-1}\left(\frac{x^2+1}{x^2-1}\right)$$

$$y = \frac{\pi}{2} \Rightarrow \frac{dy}{dx} = 0$$

**Sol.9 B**

$y = x - x^2$

$y^2 = x^2 + x^4 - 2x^3$

$u = y^2$

$u = x^2 + x^4 - 2x^3$

$\frac{du}{dx} = 2x + 4x^3 - 6x^2 = \frac{du}{dv} = 2x^2 - 3x+1$

$v = x^2 \Rightarrow dv/dx = 2x$

**Sol.10 D**

$y = f(x)$ , $f(e^x)$

$f'(e^x) . e^x \Rightarrow f''(e^x)\, e^{2x} + e^x f'(e^x)$

**Sol.11 D**

$x = at^2$, $y = 2at$

$\frac{dy}{dx} = \frac{2a}{2at} = \frac{1}{t}$

$\frac{d^2y}{dx^2} = \frac{-1}{t^2} \times \frac{dt}{dx} \Rightarrow \frac{d^2y}{dx^2} = \frac{-1}{2at^3}$

**Sol.13 D**

$y = \sin^{-1} (x\sqrt{1-x} + \sqrt{x}\sqrt{1-x^2})$

$\frac{dy}{dx} = \frac{1}{2\sqrt{x(1-x)}} + p$

$y = \sin^{-1}(x) + \sin^{-1}(\sqrt{x})$

$\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}} + \frac{1}{2\sqrt{x}\sqrt{1-x}}$

**Sol.14 C**

$\frac{d}{dx}\left(\frac{x^4+x^2+1}{x^2+x+1}\right) = ax + b$

$\frac{d}{dx}\left\{\frac{(x^2+x+1)^2 x - 2(x^3+x^2+x)}{(x^2+x+1)}\right\}$

$\frac{d}{dx}\{(x^2 + x + 1) - 2x\} = 2x - 1$

**Sol.15 B**

$$f(x) = \begin{vmatrix} \cos x & x & 1 \\ 2\sin x & x^2 & 2x \\ \tan x & x & 1 \end{vmatrix}$$

$$f'(x) = \begin{vmatrix} -\sin x & 1 & 0 \\ 2\sin x & x^2 & 2x \\ \tan x & x & 1 \end{vmatrix} + \begin{vmatrix} \cos x & x & 1 \\ 2\cos x & 2x & 2 \\ \tan x & x & 1 \end{vmatrix} + \begin{vmatrix} \cos x & x & 1 \\ 2\sin x & x^2 & 2x \\ \sec^2 x & 1 & 0 \end{vmatrix}$$

$$\frac{f'(x)}{x} = \frac{-1}{x}\begin{vmatrix} -\sin x & 1 & 0 \\ 2\sin x & x^2 & 2x \\ \tan x & x & 1 \end{vmatrix} + \frac{1}{x}\begin{vmatrix} \cos x & x & 1 \\ 2\cos x & 2x & 2 \\ \tan x & x & 1 \end{vmatrix}$$

$$+\frac{1}{x}\begin{vmatrix} \cos x & x & 1 \\ 2\sin x & x^2 & 2x \\ \sec^2 x & 1 & 0 \end{vmatrix}$$

$$= \begin{vmatrix} -\sin x & 1 & 0 \\ 2\frac{\sin x}{x} & x & 2 \\ \tan x & x & 1 \end{vmatrix} + \begin{vmatrix} \cos x & 1 & 1 \\ 2\cos x & 2 & 2 \\ \tan x & 1 & 1 \end{vmatrix} + \begin{vmatrix} \cos x & x & 1 \\ 2\frac{\cos x}{x} & x & 2 \\ \sec^2 x & 1 & 0 \end{vmatrix}$$

$$\lim_{x\to 0} \frac{f'(x)}{x} = \begin{vmatrix} 0 & 1 & 0 \\ 2 & 0 & 2 \\ 0 & 0 & 1 \end{vmatrix} + 0 + 0 = 0 - 1\,(2) + 0 = -2$$

**Sol.16 C**

$u = \cos x + b,\ \frac{d^x}{dx^n}\ \{f(ax+b)\}$

**Sol.17 B**

$y = x + e^x$

$\frac{dx}{dy} = 1 - e^x . \frac{dx}{dy}$

$\frac{dx}{dy} = \frac{1}{(1+e^x)}$

$\frac{d^2x}{dy^2} = \left\{\frac{(1+e^x)\times 0 - e^x}{(1+e^x)^2}\right\} \frac{1}{(1+e^x)}$

$= \frac{-e^x}{(1+e^x)^3}$

**Sol.18 B**

$y = f\left(\frac{2x-1}{x^2+1}\right)$, $f'(x) = \sin x$

$\frac{dy}{dx} = f'\left(\frac{2x-1}{x^2+1}\right) . \frac{2(x^2+1) - 2x(2x-1)}{(x^2+1)^2}$

$= \sin\left(\frac{2x-1}{x^2+1}\right)\left\{\frac{2x^2+2-4x^2+2x}{(x^2+1)^2}\right\}$

$$= \sin\left(\frac{2x-1}{x^2+1}\right)\left\{\frac{2(1+x-x^2)}{(x^2+1)^2}\right\}$$

**Sol.19 C**

$$8f(x) + 6f\left(\frac{1}{x}\right) = x + 5$$

$$y = x^2\, f(x)$$

$$\frac{dy}{dx} = x^2 \,.\, f'(x) + 2x\, f(x)$$

$$8f'(x) - \frac{6}{x^2}\, f'\left(\frac{1}{x}\right) = 1$$

$$8f'(-1) - 6\,f'(-1) = 1$$

$$2f'(-1) = 1 \;\Rightarrow\; f'(-1) = \frac{1}{2}$$

$$f(-1) = \frac{4}{14}$$

$$\left.\frac{dy}{dx}\right|_{x=-1} = 1 \times \frac{1}{2} - 2 \times \frac{4}{14} = \frac{7-8}{14} = \frac{-1}{14}$$

**Sol.20 C**

$$x = e^y + e^{y+e^{y+\ldots\infty}}$$

$$x = e^{y+x}$$

$$x = e^{(y+x)}\left\{\frac{dy}{dx}+1\right\}$$

$$\frac{dy}{dx} = \frac{1-e^{x+y}}{e^{x+y}} = \frac{1-x}{x}$$

**Sol.21 C**

$$f(x) = x^n$$

$$f(1) - \frac{f'(1)}{11} + \frac{f''(1)}{21} - \ldots + (+)\frac{f^n(1)}{n!}$$

$$f'(x) = n \,.\, x^{n-1} \Rightarrow f'(1) = n$$
$$f''(x) = n(n-1)\, x^{n-2} \Rightarrow f''(1) = n(n-1)$$
$$f''(x) = n(n-1)\,(n-2)\, x^{n-3}$$
$$\Rightarrow f''(1) = n(n-1)\,(n-2)$$

$$\vdots$$

$$f(1-x)^n = 1 - nx + \frac{n(n-1)x^2}{2!} - \ldots$$

$$0 = 1 - n + \frac{n(n-1)}{2!} + \ldots$$

**Sol.22 C**

$$y = \frac{a+bx^{3/2}}{x^{5/4}}$$

$$\left.\frac{dy}{dx}\right|_{x=5} = 0 \;\Rightarrow\; \left.\left(-\frac{5^{-9/4}}{4}xa+\frac{b}{4}x^{-3/4}\right)\right|_{x=5} = 0$$

$$\left(\frac{5}{4}\,.\,ax^{-9/4}\right)_{x=5} = \left(\frac{b}{4}\,x^{-3/4}\right)_{x=5}$$

$$\left(\frac{5}{4}\,.\,ax^{-6/4}\right)_{x=5} = \frac{b}{4}$$

$$\Rightarrow \quad 5a\,.\,(5)^{-6/4} = b$$

$$\frac{a}{b} = \frac{1}{5^{1-6/4}} = \frac{1}{5^{-1/2}} = \sqrt{5}$$

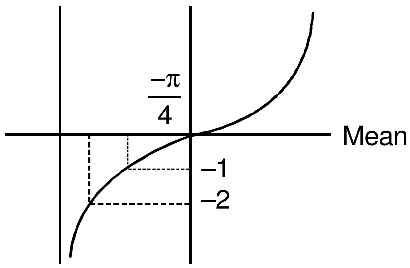
**Sol.23 B**

$f(x) = f'(x) + f''(x) + \ldots \infty$
$f(0) = 1$

**Sol.24 C**

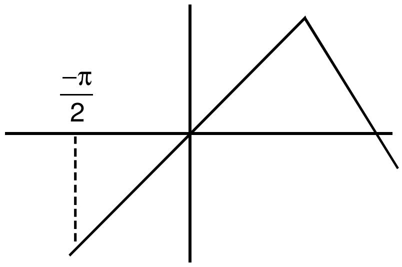
$$y = \sin^{-1}\left(\frac{2x}{1+x^2}\right) \,.\, \left.\frac{dy}{dx}\right|_{x=-2}$$

$$x = \tan\theta \quad\Rightarrow\quad y = \sin^{-1}(\sin 2\theta)$$




$$\theta < -\frac{-\pi}{4}$$

$$2\theta < -\frac{-\pi}{2}$$



$y = \pi - 2\theta = \pi - 2\tan^{-1} x$

$$\left.\frac{dy}{dx}\right|_{x=-2} = \frac{-2}{(1+x^2)} = \frac{-2}{5}$$

**Sol.25 D**

$y = \sqrt{\sin x + y}$

$y^2 = \sin x + y$

$2yy' = \cos x + y'$

$$y' = \frac{\cos x}{2y-1}$$

**Sol.26 A**

$y = e^{-x}\cos x,\ y_4 + ky = 0$

$y_1 = -e^{-x}\cos x - e^{-x}\sin x$

$y_2 = e^{-x}\sin x + e^{-x}\cos x - \{e^{-x}\cos x - e^{-x}\sin x\}$

$y_2 = 2e^{-x}\sin x$

$y_3 = 2e^{-x}\cos x - 2e^{-x}\sin x$

$y_4 = -2e^{-x}\sin x - 2e^{-x}\cos x + 2e^{-x}\sin x$
$- 2e^{-x}\cos x$

$y_4 + 4y = 0$

**Sol.27 C**

$y = a\cos \ell n\, x + b \sin \ell n\, x$

$$y' = -\frac{a}{x}\sin \ell nx + \frac{b}{x}\cos \ell nx$$

$xy' = -a\sin \ell nx + b\cos \ell nx$

$$xy'' + y' = -\frac{a\cos \ell nx}{x} - \frac{b\sin \ell n\, x}{x}$$

$x^2y'' + xy' = -y$

**Sol.28 D**

$$y = \sin mx\ ,\ \begin{vmatrix} y & y_1 & y_2 \\ y_3 & y_4 & y_5 \\ y_6 & y_7 & y_8 \end{vmatrix}$$

**Sol.29 D**

$$f'(4) = 5,\ \lim_{x\to 2}\frac{f(4)-f(x^2)}{2-x}$$

$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$f'(x)\ \lim_{x\to 2}\ 0 - \frac{f'(x^2).2x}{-1}$$

$f'(4)\ .\ 4 = 20$

**Sol.30 D**

$y = e^{2x}$

$$\frac{dy}{dx} = 2e^{2x} \quad \Bigg| \quad \frac{dx}{dy} = \frac{1}{2}e^{-2x},$$

$$\frac{d^2y}{dx^2} = 4e^{2x} \quad \Bigg| \quad \frac{d}{dy}\left(\frac{dx}{dy}\right) = -e^{-2x}\times\frac{1}{2e^{2x}}$$

$$\left(\frac{d^2y}{dx^2}\right)\left(\frac{d^2x}{dy^2}\right) = -\frac{4e^{2x}\times e^{-4x}}{2} = -2e^{-2x}$$

**Sol.31 B**

$g(x) = f^{-1}(x)$

$$f'(x) = \frac{x^5}{(1+x^4)},\ g(x) = a,\ f'(2) = ?$$

$f'(g(x)) = x\ \ g'(f(x))\ .\ f'(x) = 1$

$g'(f(x))\ .\ g'(x) = 1$

$f'(a)\ .\ g'(2) = 0$

**Sol.32 B**

$y = (1+x)(1+x^2)\ ...\ (1+x^{2n})$

$$y = \frac{(1-x^2)(1+x^2)(1+x^4)...(1+x^{2n})}{(1-x)}$$

$$y = \frac{1-x^{4n}}{1-x}$$

$$\frac{dy}{dx} = \frac{(1-x)(-4nx^{4n-1}) + (1-x^{4n})}{(1-x)^2}$$

$$\frac{dy}{dx} = \frac{-4nx^{4n-1} + 4nx^{4n} + 1 - x^{4n}}{(1-x)^2}$$

$$\left.\frac{dy}{dx}\right|_{x=0} = \frac{-4n\times 0 + 0 + 1 - 0}{1} = 1$$

**Sol.33 A**

$$u = \sec^{-1}\frac{1}{(2x^2-1)}\quad v = \sqrt{1-x^2}$$

$$u = \cos^{-1}(2x^2-1)\quad \frac{dv}{dx} = \frac{-x}{\sqrt{1-x^2}}$$

$$\frac{du}{dx} = \frac{-1\times(4x)}{\sqrt{1-(2x^2-)^2}}$$

$$\frac{du}{dv} = \frac{-4x}{\sqrt{-4x^4+4x^2}} \times \frac{\sqrt{1-x^2}}{-x} = \frac{4}{2x}$$

$$\left|\frac{du}{dv}\right|_{x=1/2} = \frac{4}{2(1/2)} = 4$$

**Sol.34 C**

$$f(x) = \begin{vmatrix} \cos x & \sin x & \cos x \\ \cos 2x & \sin 2x & 2\cos 3x \\ \cos 3x & \sin 3x & 3\cos 3x \end{vmatrix}$$

$$f'(x) = \begin{vmatrix} \sin x & \cos x & -\sin x \\ \cos 2x & \sin 2x & 2\cos 2x \\ \cos 3x & \sin 3x & 3\cos 3x \end{vmatrix}$$

$$+ \begin{vmatrix} \cos x & \sin x & \cos x \\ -2\sin 2x & 2\cos 2x & -4\sin 2x \\ \cos 3x & \sin 3x & 3\cos 3x \end{vmatrix}$$

$$+ \begin{vmatrix} \cos x & \sin x & \cos x \\ \cos 2x & \sin 2x & 2\cos 2x \\ -3\sin 3x & 3\cos 3x & -9\sin 3x \end{vmatrix}$$

$$f'\left(\frac{\pi}{2}\right) = \begin{vmatrix} -1 & 0 & -1 \\ -1 & 0 & -2 \\ 0 & -1 & 0 \end{vmatrix} + \begin{vmatrix} 0 & 1 & 0 \\ 0 & -2 & 0 \\ 0 & -1 & 0 \end{vmatrix} + \begin{vmatrix} 0 & 1 & 0 \\ -1 & 0 & -2 \\ 3 & 0 & 9 \end{vmatrix}$$

$= -1(-2) + 0 - 1(1) + 0 - 1\,(-3) + 0$

$= 2 - 1 + 3 = 4$

## EXERCISE – II

## HINTS & SOLUTIONS

**Sol.1 A,B**

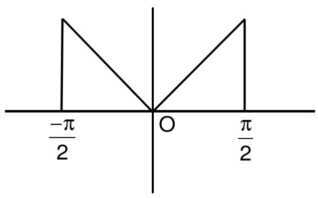
$\sin^{-1}\dfrac{t}{\sqrt{1-t^2}}$ w.r.t. $\cos^{-1}\dfrac{t}{\sqrt{1+t^2}}$

Put $t = \tan\theta - \dfrac{\pi}{2} < \theta < \dfrac{\pi}{2}$

$u = \sin^{-1}\left(\dfrac{\tan\theta}{|\sec\theta|}\right)$

$\Rightarrow u = \sin^{-1}(\sin\theta)$

$u = \theta \Rightarrow \tan^{-1} t$




$\dfrac{du}{dt} = \dfrac{1}{(1+t^2)}$

$V = \cos^{-1}\dfrac{1}{|\sec\theta|}$

$\Rightarrow \cos^{-1}(\cos\theta)$

$\theta \quad \theta > 0 \qquad\qquad -\theta,\ \theta < 0$

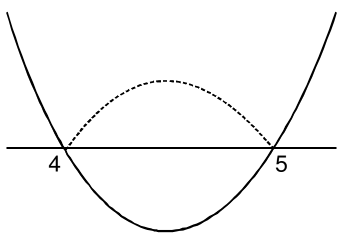
$\tan^{-1} > 0 \qquad\qquad t < 0$

$t > 0$

$V = \tan^{-1} t$

$$\left.\begin{array}{l} \dfrac{dv}{dt} = \dfrac{1}{1+t^2}, \quad t > 0 \\ \dfrac{dv}{dt} = \dfrac{1}{1+t^2}, \quad t < 0 \end{array}\right] \begin{array}{c} \text{for } t > 0, \\ 1 \\ \& \text{ for } t < 0 \\ -1 \end{array}$$

**Sol.2 B,C,D**




N.D.the for x = y & 5

$$f(x) = \begin{cases} (x-4)(x-5) & x < 4 \\ -(x-4)(x-5) & 4 < x < 5 \\ (x-4)(x-5) & x > 5 \end{cases}$$

$x > 5$

$f'(x) = 2x - 9$

$4 < x < 5$

$f'(x) = -2x + 9$

**Sol.3 A,B,D**

$x^p y^q = (x + y)^{p+q}$

$P\ell nx + q\ell ny = (p + q)\ \ell n\ (x + y)$

$\dfrac{P}{x} + \dfrac{q}{y}\ .\ y' = \dfrac{p+q}{x+y}\ (1 + y')$

$\dfrac{P}{x} + \left(\dfrac{q}{y} - \dfrac{p+q}{x+y}\right) y' = \dfrac{p+q}{x+y}$

$\dfrac{P}{x} + \left\{\dfrac{qx+qy-py-qy}{(x+y)y}\right\} y' = \dfrac{p+q}{x+y}$

$\left\{\dfrac{(qx-py)}{(x+y)y}\right\} y' = \dfrac{p+q}{x+y} - \dfrac{p}{x}$

$\left\{\dfrac{qx-py}{(x+y)y}\right\} y' = \dfrac{px+qx-px-py}{x(x+y)}$

$y' = \dfrac{(qx-py)}{(qx-py)} \times \dfrac{(x+y)y}{(x+y)x}$

$y' = \dfrac{y}{x}$

**Sol.4 A,B,C,D**

$u = e^x \sin x$ , $v = e^x \cos x$

$\dfrac{du}{dx} = e^x \cos x + e^x \sin x$

$\dfrac{dv}{dx} = e^x \cos x - e^x \sin x$

Adding : $\dfrac{du}{dx} + \dfrac{dv}{dx} = 2V = [0]$

$\dfrac{d^2u}{dx^2} = e^x \cos x - e^x \sin x + e^x \cos x + e^x \sin x$

$= 2V \Rightarrow$ [B]

$v\ .\ \dfrac{du}{dx} = u\ .\ \dfrac{du}{dx} = e^{2x}\cos^2 x + e^{2x}\sin x \cos x$

$- e^{2x}\sin x \cos x + e^{2x}\sin^2 x$

$= u^2 + v^2$

**Sol.5 B,C**

$\sqrt{x^2+y^2} = e^t$ , $t = \sin^{-1}\dfrac{y}{\sqrt{x^2+y^2}}$

$$\sqrt{x^2+y^2} = \sin^{-1}\frac{y}{\sqrt{x^2+y^2}}$$

$$\frac{1}{2\sqrt{x^2+y^2}} \times (2x + 2yy') = \frac{1}{\sqrt{1-\frac{y^2}{x^2+y^2}}}$$

$$\frac{\sqrt{x^2+y^2}\,y' - y\left(\frac{2x+2yy'}{2\sqrt{x^2+y^2}}\right)}{(x^2+y^2)}$$

$$\frac{x+yy'}{\sqrt{x^2+y^2}} = \frac{\sqrt{x^2+y^2}}{x} \times \left\{\frac{(x^2+y^2)y'xy - y^2y^1}{x^2+y^2}\right\}$$

$$\frac{x+yy'}{\sqrt{x^2+y^2}} = \frac{x^2y^1 - xy}{x\sqrt{x^2+y^2}}$$

$$x^2 + xyy' = x^2y' - xy$$

$$y' = \frac{x^2+xy}{x^2-xy} \Rightarrow \frac{x+y}{x-y}$$

**Sol.6 A,C**

$f_n(x) = e^{f_{n-1}(x)}$

$f_0(x) = x,\ \frac{d}{dx}\{f_n(x)\} = ?$

$f_1(x) = e^{f_0(x)} = e^x$

$f_2(x) = e^{f_1(x)} = e^{e^x}$

$f_3(x) = e^{f_2(x)} = e^{e^{e^x}}$

$f_n(x) = e^{f_{n-1}(x)} = e^{e^{e^{\cdots\cdots e^x}}}$

$\frac{d}{dx}(f_n(x)) = f'_{(n-1)}(x)\, e^{f_{n-1}(x)}$

$f'_{1(x)} = e^x,\ f'_{2(x)} = e^x.e^{e^x}$

$f'_3(x) = e^{e^{e^x}}.e^{e^x}.e^x = f_3(x).f'_2(x)$

$f_n(x) = f_n(x).f'_{(n-1)}x$

$f_3(x) = f_3(x).f_2(x)\ f(x)$

$f_n(x) = f_n(x)\ f_{(n-1)}x\ ..........\ f(x)$

**Sol.7 C,D**

$f''(x) = -f(x),\ f'(x) = g(x)$

$h'(x) = (f(x))^2 + (g(x))^2$

$h''(x) = 2f(x)\,f'(x) + 2g(x).g'(x)$

$f'(x) = +g(x)$

$f''(x) = g'(x) = -f(x)$

$h''(x) = -2\,g'(x).g(x) + 2\,g(x)\,g'(x)$

$h'(x) = k$

$h(x) = kx + c$

$h(0) = 2 \qquad h(1) = 4$

$c = 2 \qquad k = 2$

$h(x) = 2x + 2$

**Sol.8 B,C,D**

$$f(x) = \begin{vmatrix} \cos(x+x^2) & \sin(x+x^n) & -\cos(x+x) \\ \sin(x-x^2) & \cos(x-x^2) & \sin(x-x^2) \\ \sin 2x & 0 & \sin 2x^2 \end{vmatrix}$$

**Sol.9 A,B,C,D**

$f(x) = (ax + b)\sin x + (cx + d)\cos x$

$f'(x) = (ax + b)\cos x + a\sin x - (cx + d)\sin x + c\cos x$

$x\cos x = (ax + b)\cos x + a\sin x - (cx + d)\sin x + c\cos x$

$a = 1,\ d = 1,\ b = 0,\ c = 0$

**Sol.10 B,C**

$$y = \cos^1\sqrt{\frac{\sqrt{1+x^2}+1}{2\sqrt{1+x^2}}}$$

put $x = \tan\theta - \frac{\pi}{2} < \theta < \frac{\pi}{2}$

$$y = \cos^{-1}\sqrt{\frac{1+\cos\theta}{2}} = \cos^{-1}\left(\cos\frac{\theta}{2}\right)$$

$-\frac{\theta}{2},\ x < 0 \qquad \frac{\theta}{2},\ x > 0$

$$\frac{dy}{dx} = -\frac{1}{2(1+x)},\ x < 0,\ \frac{1}{2(1+x)},\ x > 0.$$

**Sol.11 A,B,C**

$f, g, f(0) = \frac{2}{g(0)}$

$f'(0) = 2g'(0) = 4g(0),\ g''(0) = 5f''(0) = g(0) = 3$

(A) $h(x) = \frac{f(x)}{g(x)} \Rightarrow h'(x) = \frac{f'(g) - g'f}{g^2}$

$$= h'(0) = \frac{f'(0)\,g(0) - g'(0)\,f(0)}{g^2(0)}$$

$$= h'(0) = \frac{4(g(0))^2 - 2g(0).\frac{2}{g(0)}}{9}$$

$$h'(0) = \frac{36-4}{9} = \frac{32}{5} \qquad (A)$$

k(x) = f(x). g(x). sin x

k′(x) = f(x) g(x) sin x + f(x) g′(x) sin x + f′(x) g(x) sin x

k′(x) = 2

$$\lim_{x\to 0} \frac{g'(x)}{f'(x)} = \lim_{x\to 0} \frac{g'(x)}{f'(x)} = \frac{1}{2}$$

**Sol.12 A,B**

$$y = \tan^{-1}\left\{\frac{\ell n\left(\frac{e}{x^2}\right)}{\ell n\,(ex^2)}\right\} + \tan^{-1}\left(\frac{3+2\ell n\,x}{1-6\,\ell n\,x}\right)$$

$$y = \tan^{-1}\left(\frac{1-2\ell n\,x}{1+2\ell n\,x}\right) + \tan^{-1}\frac{3+2\ell n\,x}{1-6\ell n\,x}$$

$$\tan y = \frac{\frac{1-2\ell n\,x}{1+2\ell n\,x} + \frac{3+2\ell n\,x}{1-6\ell n\,x}}{1-\frac{(1-2\ell n\,x)\,(3+2\ell n\,x)}{(1+2\ell n\,x)\,(1-6\ell n\,x)}}$$

$$= \frac{4+16(\ell n\,x)^2}{-2-8(\ell n\,x)^2} \Rightarrow -\frac{(2+8(\ell n\,x)^2)}{1+4(\ell n\,x)^2}$$

$$\Rightarrow -2$$

$$\frac{dy}{dx} = 0 \Rightarrow \frac{d^2y}{dx} = 0$$

## EXERCISE – III HINTS & SOLUTIONS

**Sol.1 (i)** $f(x) = \sin x^2 = y$

$y + \Delta y = \sin (x + \Delta x)^2$

$\Delta y = \sin (x + \Delta x)^2 - \sin x^2$

$$\lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x} = \lim_{\Delta x\to 0}\frac{\sin(x+\Delta x)^2-\sin x^2}{\Delta x}$$

$$\frac{dy}{dx} = \lim_{\Delta x\to 0}\frac{\sin(x+\Delta x)^2-\sin x^2}{\Delta x}$$

$$\frac{dy}{dx} = \lim_{\Delta x\to 0}\frac{\sin(x^2+\Delta x^2+\ell x\Delta x)-\sin x^2}{\Delta x}$$

$$= \lim_{\Delta x\to 0}$$

$$\frac{2\sin\left(\frac{\Delta x^2+2x\Delta x}{2}\right)\cos\left(\frac{2x^2+\Delta x^2+2x\Delta x}{2}\right)}{\Delta x}$$

$$= \lim_{\Delta x\to 0}\frac{2\sin\Delta x\left(\frac{\Delta x}{2}+x\right)\cos\left(x^2+x\Delta x+\frac{\Delta x^2}{2}\right)}{\dfrac{\Delta x\left(x+\frac{\Delta x}{2}\right)}{\left(\frac{x+\Delta x}{2}\right)}}$$

$$= \lim_{\Delta x\to 0} 2\left(x\frac{\Delta x}{2}\right)\cos\left(x^2+x\Delta x+\frac{\Delta x^2}{2}\right)$$

$= 2x \cos x^2$

**(ii)** $f(x) = e^{2x+3} = y$

$y + \Delta y = e^{(2x+\Delta x)\ +\ 3}$

$$\lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x} = \lim_{\Delta x\to 0}\frac{e^{2(x+\Delta x)+3}-e^{2x+3}}{\Delta x}$$

$$\frac{dy}{dx} = \lim_{\Delta x\to 0} e^{2x+3}\left\{\frac{(e^{2\Delta x}-1)2}{2\Delta x}\right\}$$

$$\frac{dy}{dx} = 2.e^{(2x+3)}$$

**Sol.2** **(i)** $y = x^{2/3} + 7e - \dfrac{5}{x} + 7\tan x$

$$\frac{dy}{dx} = \frac{2}{3}x^{-1/3} + \frac{5}{x^2} + 7\sec^2 x$$

**(ii)** $y = x^2 . \ell nx . e^x$

$$\frac{dy}{dx} = x^2\ell nx.\ e^x + xe^x + 2xe^x\ \ell nx$$

**(iii)** $y = \ell n \tan\left(\dfrac{\pi}{4}+\dfrac{x}{2}\right)$

$$\frac{dy}{dx} = \frac{\sec^2\left(\frac{\pi}{4}+\frac{x}{2}\right)}{\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)} \times \frac{1}{2}$$

$$= \frac{1}{\sin\left(\frac{\pi}{2}+x\right)} \Rightarrow \sec x$$

**(iv)** $y = \dfrac{\sin x - x\cos x}{x\sin x+\cos x}$

$$y = \frac{\tan x - x}{x\tan x + 1}$$

$$y' = \frac{(x\tan x+1)(\sec^2 x-1)-(\tan x-x)(x\sec^2 x+\tan x)}{(x+\tan x+1)^2}$$

$$= \frac{\begin{matrix} x\tan x\sec^2 x - x\tan x + \sec^2 x - 1 - x\sec^2 x\tan x \\ -\tan^2 x + x^2\sec^2 x + x\tan x\end{matrix}}{(x\tan x+1)^2}$$

$$\frac{dy}{dx} = \frac{x^2\sec^2 x}{(x\tan x+1)^2}$$

$$\frac{dy}{dx} = \frac{x^2}{(x\sin x+\cos x)^2}$$

**(v)** $y = \tan\left(\tan^{-1}\sqrt{\dfrac{1-\cos x}{1+\cos x}}\right)$

$$y = \tan\left\{\tan^{-1}\sqrt{\frac{2\sin^2\frac{x}{2}}{2\cos^2\frac{x}{2}}}\right\}$$

$$y = \tan\left\{\tan^{-1}\left(\tan\frac{x}{2}\right)\right\}$$

$$\frac{dy}{dx} = \frac{1}{2}\sec^2\left(\frac{x}{2}\right)$$

**Sol.3** $f(x) = 2\,\ell n\,(x-2) - x^2 + 4x + 1$

$$f'(x) = \frac{2}{(x-2)} - 2x + 4$$

$$f'(x) \geq 0 \Rightarrow \frac{2-2x^2+4x+4x}{(x-2)} \geq 0$$

$$\frac{-2x^2+8x-6}{(x-2)} \geq 0 \;\Rightarrow\; \frac{x^2-4x+3}{(x-2)} \leq 0$$

$$\Rightarrow \frac{(x-3)(x-1) \leq 1}{(x-2)} \Rightarrow x \in (2,\,3]$$

**Sol.4** **(i)** $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$

$$2ax + 2hx\frac{dy}{dx} + 2hy + 2by\frac{dy}{dx} + 2g + 2f\frac{dy}{dx} = 0$$

$$\frac{dy}{dx}(2hx+2by+2f) = -2ax - 2hy - 2g$$

$$\frac{dy}{dx} = \frac{-2ax-2hy-2g}{2hx+2by+2f} = -\frac{(ax+hy+g)}{(hx+by+f)}$$

**(ii)** $xy + xe^{-y} + y.e^x = x^2$

$$x\frac{dy}{dx} + y + xe^{-y}\left(\frac{-dy}{dx}\right) + e^{-y} + ye^x + \frac{e^x dy}{dx} = 2x$$

$$\frac{dy}{dx}(x - xe^{-y} + e^x) = 2x - y - e^{-y} - ye^x$$

$$\frac{dy}{dx} = \frac{2x-y-e^{-y}-ye^x}{x(1-e^{-y})+e^x}$$

**Sol.5** **(i)** $y = (\ell n\,x)\cos x$

$\ell ny = \cos x\;\ell n\,(\ell n\,x)$

$$y' = (\ell n\,x)^{\cos x}\left\{\frac{\cos x}{x\ell nx} + \ell n(\ell nx)(-\sin x)\right\}$$

**(ii)** $y = x^x - 2^{\sin x}$

$$\frac{dy}{dx} = x^x(1+\ell nx) - 2^{\sin x}\,\ell n2 \times \cos x$$

**(iii)** $y = (x\,\ell nx)^{\ell n(\ell nx)}$

$\ell ny = \ell n(\ell nx)\,.\,\ell n\,(x\ell nx)$

$$\frac{dy}{dx} = (x\ell nx)^{\ell n(\ell nx)}\left\{\ell n(x\ell nx)\times\frac{1}{x\ell nx} + \ell n(\ell nx)\times\frac{1}{x\ell nx}(\ell nx+1)\right\}$$

$$= (xnx)^{\ell n(\ell nx)}\left\{\frac{\ell n(x\ell nx)}{(x\ell nx)} + \frac{\ell n(\ell nx)}{x} + \frac{\ell n(\ell nx)}{x\ell nx}\right\}$$

**Sol.6** $Pn = a\left(\frac{1-r^n}{1-r}\right)$

$$\frac{dP_n}{dr} = a\left\{\frac{(1-r)(-nr^{n-1})+(1-r^n)}{(1-r)^2}\right\}$$

$$\frac{dPn}{dr} = a\left\{\frac{-nr^{n-1}+nr^n+1-r^n}{(1-r)^2}\right\}$$

$$\frac{dPn}{dr} = a\left\{\frac{n(1-r^{n-1})+n(r^n-1)+(1-r^n)}{(1-r)^2}\right\}$$

$$\frac{dPn}{dr} = \frac{na(1-r^{n-1})}{(1-r)^2} - \frac{a(1-r^n)}{(1-r)^2}\;(n+1)$$

$$(1-r)\frac{dPn}{dr} = nP_{n-1} - (n-1)\,Pn$$

**Sol.7** $x = at^3,\; y = bt^2$

$$\frac{dx}{dt} = 3at^2 \;\&\; \frac{dy}{dt} = 2bt$$

$$\frac{dy}{dx} = \frac{2b}{3at} \Rightarrow \frac{d^2y}{dx^2} = \frac{-2b}{3at^2} \times \frac{1}{3at^2}$$

$$\frac{d^2y}{dx^2} = \frac{-2b}{9a^2+4} \Rightarrow \frac{d^3y}{dx^3} = \frac{8b}{9a^2t^5} \times \frac{1}{3at^2}$$

$$d^3y/dx^3 = \frac{8b}{27a^3+7}$$

**Sol.8** $y = \ell n\tan\left(\frac{x}{2}\right)$

$$\frac{d^2y}{dx^2} + \cot x\,\frac{dy}{dx} + 4y\,\mathrm{cosec}^2x = 0$$

$$\frac{dz}{dx} = \frac{1}{\tan\left(\frac{x}{2}\right)} \times \frac{1}{2}\sec^2\frac{x}{2} \Rightarrow \mathrm{cosec}x$$

$$\frac{dy}{dx} = \frac{dy}{dz}\cdot\frac{dz}{dx}$$

$$\frac{dy}{dx} = \left(\frac{dy}{dz}\right) \text{cosecx}$$

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left\{(\text{cos ecx}).\frac{dy}{dz}\right\}$$

$$= -\text{ cosecx cot x}\,\frac{dy}{dz} + \text{cosecx}\,\frac{d}{dx}\left(\frac{dy}{dz}\right)$$

$$= -\text{ cosecx cotx}\,\frac{dy}{dz} + \text{cosecx}\,\frac{d^2y}{dz^2} \times \text{cosecx}$$

$$-\text{cosec x cotx}\,\frac{dy}{dz} + \text{cosec}^2\text{x}\,\frac{d^2y}{dz^2} + \text{cotx}$$

$$\text{cosecx}\frac{dy}{dz} + 4y\,\text{cosec}^2x = 0$$

$$\frac{d^2y}{dz^2} + 4y = 0$$

**Sol.9** $f(x) = x^n$

$$f(1) + \frac{f'(1)}{1!} + \frac{f^2(1)}{2!} + \ldots + \frac{f^n(1)}{n!}$$

$f'(x) = nx^{n-1}$

$f^2(x) = n(n-1)\,x^{n-2}$

$$(1 + x)^n = 1 + nx + \frac{n(n-1)x^2}{2!} + \ldots$$

$$2^n = 1 + n + \frac{n(n-1)}{2!} + \ldots.$$

$$2^n = 1 + f'(1) + \frac{f^2(1)}{2!} + \ldots$$

**Sol.10** $\displaystyle\lim_{x\to 0} \frac{a\sin x - bx + cx^2 + x^3}{2x^2.\ell n(1+x) - 2x^3 + x^4}$

$$\lim_{x\to 0} \frac{a\cos x - b + 2cx + 3x^2}{4x\ell n(1+x) + \frac{2x^2}{1+x} - 6x^2 + 4 + x^3}$$

$$\frac{a-b}{0} \Rightarrow \quad a = b$$

$$\lim_{x\to 0} \frac{a\cos x - b + 2cx + 3x^2}{4x\ln(1+x) + \frac{2x^2}{1+x} - 6x^2 + 4x^3}$$

For existence c = 0

$$\text{So, } \lim_{x\to 0} \frac{-\frac{a}{2}+3}{4+x-6} \Rightarrow \frac{-a+6}{-20}$$

a = 6 , b = 6 , c = 0

$$\lim_{x\to 0} \frac{-6(x-\sin x)+x^3}{2x^2\ell n(1+x) - 2x^3 + x^4}$$

$$\lim_{x\to 0} \frac{-6\times\frac{1}{6}+1}{2\times 1 - 2 + x}$$

**Sol.11** Take ln on both the side

$$\ln\cos\frac{x}{2} + \ln\cos\frac{x}{2^2} + \ldots\ldots.. = \ln\sin x - \ln x$$

Diff. w.r.t. x

$$\frac{-1}{2}\tan\frac{x}{2} - \frac{1}{2^2}\tan\frac{x}{2^2} \;\ldots\ldots. = \cot x - \frac{1}{x}$$

Again diff. w.r.t. x

$$\frac{-1}{2}\sec^2\frac{x}{2} - \frac{1}{2^4}\sec^2\frac{x}{2^2} + \ldots. = -\text{cosec}^2x + \frac{1}{x^2}$$

$$\frac{1}{2^2}\sec^2\frac{x}{2} + \frac{1}{2^4}\sec^2\frac{x}{2^2} + \ldots\ldots = \text{cosec}^2x - \frac{1}{x^2}$$

**Sol.12** $x = e^t \sin t$ , $y = e^t \cos t$

$y^4\,(x + y)^2 = 2(xy' - y)$

$$\frac{dx}{dt} = e^t\cos t + e^t \sin t,\quad \frac{dy}{dt} = -e^t\sin t + e^t\cos t$$

$$\frac{dy}{dx} = \frac{\cos t - \sin t}{\cos t + \sin t} = y'$$

$$y'' = \left\{\frac{(\cos t + \sin t)(-\sin t - \cos t) - (\cos t - \sin t)(\cos t - \sin t)}{(\cos t + \sin t)^2}\right\}\frac{dt}{dx}$$

$$y'' = \frac{-2}{e^t(\cos t + \sin t)^3}$$

$(x + y)^2 = e^{2t}\,(1 + \sin 2t)$

$$y''(x + y)^2 = \frac{-2e^t(1+\sin 2t)}{(\cos t + \sin t)^3} = \frac{-2et}{(\sin t + \cos t)}$$

$$2(xy' - y) = 2\left\{e^t \sin t\left(\frac{\cos t - \sin t}{\cos t + \sin t}\right) - e^t \cos t\right\}$$

$$2(xy' - y) = 2\left[\frac{e^t \sin t \cos t - e^t \sin^2 t - e^t \cos^2 t - e^t \sin t \cos t}{(\cos t + \sin t)}\right]$$

$$= \frac{-2e^t}{(\cos t + \sin t)}$$

LHS = RHS

**Sol.13** $y = x \log\left(\frac{x}{a + bx}\right)$

$$\frac{dy}{dx} = \left\{\frac{(a+bx) - bx}{(a+bx)^2}\right\} + \log\left(\frac{x}{a+bx}\right)$$

$$\frac{dy}{dx} = \frac{a}{(a+bx)} + \log\left(\frac{x}{a+bx}\right)$$

$$\frac{d^2y}{dx^2} = \frac{-ab}{(a+bx)^2} + \frac{a+bx}{x}\left\{\frac{a+bx-bx}{(a+bx)^2}\right\}$$

$$\frac{d^2y}{dx^2} = \frac{-ab}{(a+bx)^2} + \frac{a}{x(a+bx)}$$

$$\frac{d^2y}{dx^2} = \frac{a}{(a+bx)}\left[\frac{1}{x} - \frac{b}{a+bx}\right]$$

$$\frac{d^2y}{dx^2} = \frac{a^2}{x(a+bx)^2}$$

$$\left.\begin{array}{l} x^3 \dfrac{d^2y}{dx^2} = \dfrac{a^2x^2}{(a+bx)^2} \\ x\dfrac{dy}{dx} - y = \dfrac{xa}{a+bx} \\ \left(x\dfrac{dy}{dx} - y\right)^2 = \dfrac{a^2x^2}{(a+bx)^2} \end{array}\right] \text{LHS=RHS}$$

**Sol.14** $y = (\cos x)^{\ell nx} + (\ell nx)^x$

$u = (\cos x)^{\ell nx}$

$\ell nu = \ell nx \,.\, (\ell n \cos x)$

$$\frac{du}{dx} = \left[\ell nx\,(-\tan x) + \frac{\ell n \cos x}{x}\right](\cos x)^{\ell n\, x}$$

$v = (\ell n\, x)^x$

$\ell nv = x\ell n\,(\ell nx)$

$$\frac{dv}{dx} = (\ell nx)^x \left\{\frac{1}{\ell nx} + \ell n(\ell nx)\right\}$$

$$\frac{dy}{dx} = \frac{dy}{dx} + \frac{dv}{dx}$$

**Sol.15** **(a)** $f(x) = \tan\{\sin^{-1}(2x)\}$

$$-\frac{\pi}{2} < \sin^{-1} 2x < \frac{\pi}{2}$$

$-1 < 2x < 1$

$$\frac{-1}{2} < x < \frac{1}{2} \quad \Rightarrow \quad x \in \left(\frac{-1}{2}, \frac{1}{2}\right)$$

$$\frac{-\pi}{2} < \sin^{-1} \sin 2x < \frac{\pi}{2}$$

$-\infty < f(x) < 0$

**(b)** $f(x) = \tan\left(\tan^{-1} \frac{2x}{\sqrt{1 - 4x^2}}\right)$

$$f(x) = \frac{2}{x\sqrt{1 - 4x^2}}$$

**(c)** $f'(x) = \sec^2 \sin^{-1}(2x) \times \frac{2}{\sqrt{1 - 4x^2}}$

$$f'\left(\frac{1}{4}\right) = \left[\sec^2\left(\sin^{-1}\frac{1}{2}\right)\right] \times \frac{2}{\sqrt{1 - \frac{1}{4}}}$$

$$= \sec^2\left(\frac{\pi}{6}\right) \times \frac{4}{\sqrt{3}} = \frac{4 \times 4}{3\sqrt{3}} \Rightarrow \frac{16\sqrt{3}}{9}$$

**Sol.16** $2y = \frac{1}{2x + (y - x)} + y - x + 2x$

$$y = \frac{1}{y + x} + x$$

$y^2 - x^2 = 0 \Rightarrow yy^1 - x = 0$

$f(100).\ f^1(100) = 100$

**Sol.17** put $x^2 = \cos 2\theta$

$$y = \tan\left(\frac{\pi}{4} + \theta\right) \; ; \; z = \sin 2\theta$$

$$\frac{dy}{dz}=\frac{\sec^2(\pi/4+\theta)}{2\cos 2\theta}=\frac{1}{(1-\sin 2\theta)\cos 2\theta}=\frac{1}{x^2(1-\sqrt{1-x^4})}$$

**Sol.18** $y = \frac{(\log_{\cos x}\sin x)}{\log_{\sin x}\cos x} + \sin^{-1}\frac{2x}{(1+x^2)}$

$$y = \left(\frac{\log\sin x}{\log\cos x}\right)^2 + \sin^{-1}\left(\frac{2x}{1+x^2}\right)$$

$$y = (\log_{\cos x}\cos x)^2 + \sin^{-1}\left(\frac{2x}{1+x^2}\right)$$

$$y' = 2\left(\frac{\log\sin x}{\log\cos x}\right)\left\{\frac{\cot x\log\cos x+\tan x\log\sin x}{(\log\cos x)^2}\right\}+\frac{dv}{dx}$$

$v = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$, $x = \tan\theta$

$v = \sin^{-1}(\sin 2\theta)$

$v = 2\theta \Rightarrow \frac{dv}{dx} = \frac{2}{1+x^2}$

$$y'\Big|_{x=\frac{\pi}{4}} = 2\left\{\frac{1\times\log\frac{1}{\sqrt2}+1\log\frac{1}{\sqrt2}}{\left(\log\frac{1}{\sqrt2}\right)^2}\right\}+\frac{2}{1+\left(\frac{\sqrt2}{4}\right)}$$

$$= \frac{4}{\frac{1}{2}\log\frac{1}{2}} + \frac{2}{1+\left(\frac{\pi}{4}\right)^2}$$

$$y' = \frac{8}{-\ell n2} + \frac{32}{11+\pi^2}$$

**Sol.19** $\sqrt{1-x^6} + \sqrt{1-y^6} = a^3(x^3-y^3)$

$x^3 = \sin A$ , $y^3 = \sin B$

$\sqrt{1-\sin^2 A} + \sqrt{1-\sin^2 B} = a^3(\sin A - \sin B)$

$\cos A + \cos B = a^3(\sin A - \sin B)$

$$2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$

$$= 2a^3\sin\left(\frac{A-B}{2}\right)\cos\left(\frac{A+B}{2}\right)$$

$$a^3 = \cot\left(\frac{A-B}{2}\right)$$

$$\cot^{-1}a^3 = \frac{A-B}{2}$$

$$\cot^{-1}a^3 = \frac{\sin^{-1}x^3}{2} - \frac{\sin^{-1}y^3}{2}$$

$$0 = \frac{3x^2}{2\sqrt{1-x^6}} - \frac{3y^2\frac{dy}{dx}}{2\sqrt{1-y^6}}$$

$$\frac{x^2}{y^2}\sqrt{\frac{1-y^6}{1-x^6}} = \frac{dy}{dx}$$

**Sol.20** $y = x + \cfrac{1}{x+\cfrac{1}{x+\cfrac{1}{x+\dots}}}$

$$y = x + \frac{1}{y}$$

$y^2 = xy + 1$

$2yy' = xy' + y \Rightarrow y' = \frac{y}{2y-x}$

$$y' = \frac{1}{2-\frac{x}{y}}$$

$$y' = \cfrac{1}{2-\cfrac{x}{x+\cfrac{1}{x+\dots}}}$$

**Sol.21** $y = \tan^{-1}\frac{u}{\sqrt{1-u^2}}$ & $x = \sec^{-1}\frac{1}{\sqrt{2u^2-1}}$

$$u \in \left(0,\frac{1}{\sqrt2}\right) \cup \left(\frac{1}{\sqrt2},1\right)$$

$u = \sin\theta$ $\quad o < u < \frac{1}{\sqrt2}$ OR $\frac{1}{\sqrt2} < u < 1$

$\left(\frac{\pi}{4},\frac{\pi}{2}\right)$ OR $\left(0,\frac{\pi}{4}\right) \rightarrow 1^{st}$ $Q^{nt}$

$$y = \tan^{-1}\left(\frac{\sin\theta}{\cos\theta}\right) = \theta$$

$$x = \sec^{-1} = \sec^{-1}\left(-\frac{1}{\cos 2\theta}\right)$$

$\Rightarrow$ $\sec^{-1}(-\sec 2\theta)$

$x = \pi - 2\theta$

$$\frac{dy}{dq} = 1, \quad \frac{dx}{dq} = -2$$

$$\frac{dy}{dx} = -\frac{1}{2} \Rightarrow 2.\frac{dy}{dx} + 1 = 0$$

**Sol.22** $y = \cot^{-1}\left(\dfrac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right)$

$$y = \cot^{-1}\left\{\frac{\left(\sin\frac{x}{2}+\cos\frac{x}{2}\right)+\left(\sin\frac{x}{2}-\cos\frac{x}{2}\right)}{\left(\sin\frac{x}{2}+\cos\frac{x}{2}\right)-\left(\sin\frac{x}{2}-\cos\frac{x}{2}\right)}\right\}$$

$$y = \cot^{-1}\left\{\frac{2\sin\frac{x}{2}}{2\cos\frac{x}{2}}\right\}$$

$$y = \cot^{-1}\left\{\cot\left(\frac{\pi}{2}-\frac{x}{2}\right)\right\}$$

$$x \in \left(0,\frac{\pi}{2}\right) \cup \left(\frac{\pi}{2},\pi\right)$$

$$-\frac{x}{2} \in \left(\frac{-\pi}{2},\frac{-\pi}{4}\right) \cup \left(\frac{-\pi}{4},0\right)$$

$$\left(\frac{\pi}{2}-\frac{x}{2}\right) \in \left(0,\frac{\pi}{4}\right) \cup \left(\frac{\pi}{4},\frac{\pi}{2}\right) \rightarrow 1^{st} \text{ quf.}$$

So, $y = \dfrac{\pi}{2} - \dfrac{x}{2}$

$$\frac{dy}{dx} = \frac{-1}{2}$$

$$y = \cot^{-1}\left\{\frac{\left(\cos\frac{x}{2}+\sin\frac{x}{2}\right)+\cos\frac{x}{2}-\sin\frac{x}{2}}{\cos\frac{x}{2}+\sin\frac{x}{2}+\sin\frac{x}{2}-\cos\frac{x}{2}}\right\}$$

$$y = \cot^{-1}\left(\cot\frac{x}{2}\right) \Rightarrow y = \frac{x}{2}$$

$$\frac{dy}{dx} = \frac{1}{2}$$

**Sol.23** $y = \tan^{-1}\dfrac{x}{1+\sqrt{1-x^2}} + \sin\left(2\tan^{-1}\sqrt{\dfrac{1-x}{1+x}}\right)$

$x \in (-1,1)$

Let $x = \cos\theta$, $\theta \in [0, \pi]$

$$y = \tan^{-1}\left(\frac{\cos\theta}{1+\sin\theta}\right) + \sin\left\{2\tan^{-1}\left(\sqrt{\frac{1-\cos\theta}{1+\cos\theta}}\right)\right\}$$

$$y = \tan^{-1}\left\{\frac{\sin\left(\frac{\pi}{2}-\theta\right)}{1+\cos\left(\frac{\pi}{2}-\theta\right)}\right\} + \sin\left\{2\tan^{-1}\left(\frac{\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}}\right)\right\}$$

$$y = \tan^{-1}\left\{\tan\left(\frac{\pi}{4}-\frac{\theta}{2}\right)\right\} + \sin\left\{2\tan^{-1}\left(\tan\frac{\theta}{2}\right)\right\}$$

$$-\frac{\pi}{2} < -\frac{\theta}{2} < 0$$

$$\frac{-\pi}{4} < \frac{\pi}{4} - \frac{\theta}{2} < \frac{\pi}{4}$$

**Sol.24** **(a)** $f(x) = x^2 - 4x - 3$, $x > 2$

$$g'(f(x)) = \frac{1}{f'(x)} = \frac{1}{(2x-4)}$$

$$g'(y) = \frac{1}{(2x-4)} = x^2 - 4x - 5 = 0$$

$(x-5)(x+1) = 0$

$x = -1, 5$

$x = 4$

$$g'(2) = \frac{1}{6}$$

**(b)** $f(0) = 1$, $g(0) = 2$, $h(0) = 3$

$(fg)'(0) = 6$, $(gh)'(0) = 4$, $(hf)'(0) = 5$

$$(fgh)'(0) = \frac{h(0)(fg')(0) + f(0)(gh)'(0) + g(0)(hf)'(0)}{2}$$

$$= \frac{3\times6+1\times4+2\times5}{2} = \frac{18+4+10}{2}$$

**Sol.25** $x = 2\cos t - \cos 2t$

$y = 2\sin t - \sin 2t$

$$\frac{dx}{dt} = -2\sin t + 2\sin 2t$$

$$\frac{dy}{dt} = 2\cos t - 2\cos 2t$$

$$\frac{dy}{dx} = \frac{\cos t - \cos 2t}{\sin 2t - \sin t}$$

$$\frac{d^2y}{dx^2} = \left[\frac{(\sin 2t - \sin t)(-\sin t + 2\sin 2t) - (\cos t - \cos 2t)(2\cos 2t - \cos t)}{(\sin 2t - \sin t)^2}\right]$$

$$\times \frac{1}{(2\sin 2t - 2\sin t)}$$

$$= \left[\frac{-\sin t\sin 2t + 2\sin^2 2t + \sin^2 t - 2\sin 2t\sin t - 2\cos t a n t + \cos^2 t + 2\cos^2 2t - \cos t\cos^2 t}{2(\sin 2t - \sin t)^3}\right]$$

$$= \frac{2 + 1 - 3(\sin 2t - \sin 2t + \cos 2t - \cos t)}{2(\sin 2t - \cos t)^3}$$

$$\frac{d^2y}{dx^2} = \frac{3(1-\cos t)}{2(\sin 2t - \sin t)^3}$$

$$\left.\frac{d^2y}{dx^2}\right|_{t=\frac{\pi}{2}} = \frac{3}{2(-1)^3} = \frac{-3}{2}$$

**Sol.26** $f(x) = x^3 + x^2 f'(1) + xf''(2) + f'''(3)$

$$\left.\begin{array}{l} f'(x) = 3x^2 + 2xf'(1) + f''(2) \\ f''(x) = 6x + 2f'(1) \\ f'''(x) = 6 \end{array}\right\} \Rightarrow \begin{array}{l} f'(1) = -5 \\ f''(2) = 2 \\ f''(3) = 6 \end{array}$$

$f(2) = 8 + 4 \times 5 + 2 \times 2 + 6 = -2$

$f(1) = 1 + 1 \times -5 + 1 \times 2 + 6 = 4$

$f(0) = f'''(3) = 6$

$f(2) = -2,\ f(1) = f(0) = 4 - 6 = -2$

So, $f(2) = f(1) - f(0)$

**Sol.27** $y = x\,\ell n\left(\frac{1}{ax} + \frac{1}{a}\right)$

$$y = x\ell n\left(\frac{a + ax}{a^2x}\right) \Rightarrow x\ell n\left(\frac{1+x}{ax}\right)$$

$$y' = \ell n\left(\frac{1+x}{ax}\right) + x\left(\frac{ax}{(1+x)}\right) \times \left\{\frac{-1}{ax^2} + 0\right\}$$

$$y' = \ell n\left(\frac{1+x}{ax}\right) - \frac{1}{(x+1)}$$

$$y'' = \frac{ax}{(1+x)} \times \left\{\frac{-1}{ax^2}\right\} + \frac{1}{(x+1)^2}$$

$$y'' = \frac{-ax(x+1) + ax^2}{ax^2(x+1)^2}$$

$$y'' = \frac{-ax}{ax^2(x+1)^2}$$

$$x(x+1)y'' = \frac{-1}{(x+1)}$$

$$y - 1 - x\frac{dy}{dx} = x\ell n\left(\frac{1+x}{ax}\right) - 1 - x\ell n\left(\frac{1+x}{ax}\right) + \frac{1}{(x+1)}$$

$$= \frac{-1}{(x+1)}$$

**Sol.28** $f(x) = \begin{cases} g(x) & x \le 0 \\ \left(\left(\frac{x+1}{x+1}\right)\right)^{\frac{1}{x}} & x > 0 \end{cases}$

Let $g(x) = ax + b$

f is cont.

$\Rightarrow f(0^+) = f(0^-) \Rightarrow b = 0$

& $y^1(1) = \frac{2}{3}\left[\ell n\left(\frac{3}{2}\right) + \frac{1}{6}\right]$

$f^1(1) = f(-1) \Rightarrow a = \frac{-2}{3}\left[\ell n\left(\frac{3}{2}\right) + \frac{1}{6}\right]$

**Sol.29** $\sin y = x(\sin a\cos y + \cos a\sin y)$

$\sin y(1 - x\cos a) = x\sin a\cos y$

$$\tan y = \frac{x\sin a}{1 - x\cos a}$$

$$y' = \frac{1}{1+\frac{x^2\sin^2 a}{(1-x\cos a)^2}} \times \frac{(1-x\cos a)\sin a - x\sin a(-\cos a)}{(1-x\cos a)^2}$$

$$y' = \frac{1}{(1+x^2-2x\cos a)} \times \sin a$$

**Sol.30** $y = \tan^{-1}(x+1) - \tan^{-1} x$

$+ \tan^{-1}(x+2) - \tan^{-1}(x+1)$

$+ \tan^{-1}(x+n) - \tan^{-1}(x+(n-1))$

$y = \tan^{-1}(x+n) - \tan^{-1} x$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{1+(x+n)^2} - \frac{1}{1+x^2}$$

# EXERCISE – IV HINTS & SOLUTIONS

**Sol.1** $|a_1 \sin x + a_2 \sin 2x + ... + a_n \sin nx| \le |\sin x|$

$\forall x \in R$

$|a_1 + 2a_1 + 3a_3 + ... + na_n| \le 1$

Differents both sides :

$|a_1 \cos x + 2a_2 \cos 2x + ... + na_n \cos nx| \le |\cos x|$

at x = 0,

$|a_1 + 2a_2 + ... + na_n| \le 1$

**Sol.2** $f(x^2) \cdot f''(x) = f'(x) \cdot f'(x^2)$ , $f(1) = 1$

$f'(1) + f''(1) = ?$, $f'''(1) = 8$

$f(x^2) . f'''(x) + 2xf'(x^2) . f''(x) = f'(x) . 2xf''(x^2) + f''(x) . f'(x^2)$

$f(1) . f'''(1) + 2. f'(1) . f''(1) = 2f'(1) f''(1) + f''(1) . f''(1)$

$f''(1) . f'(1) = 8$ ... (1)

$f(1) . f''(1) = f'(1) . f'(1)$

$$f''(1) = \frac{f'(1)^2}{f(1)} = f'(1)^2 \quad ...(2)$$

From (1) & (2), f(1) = 2

& f″ (1) = 4

So, f′(1) + f″(1) = 4 + 2 = 6

**Sol.3** $y = x \sin kx$ , $\frac{d^2y}{dx^2} + y = 2k\cos kx$

$$\frac{dy}{dx} = kx \cos kx + \sin kx$$

$$\frac{d^2y}{dx^2} = -k^2x \sin kx + k \cos kx + k \cos kx$$

$$\frac{d^2y}{dx^2} + k^2y = 2k\cos kx$$

$k^2 = 1 \Rightarrow k = \pm 1$

But 'O' is also satisfying the relation

$y = 0$, $\frac{d^2y}{dx^2} = 0$, $y = 0$, $2k \cos kx = 0$

**Sol.4** $f(x) = \frac{\sin x}{x}$, $x \neq 0$, $f(0) = 1$

$f'(x) = ?$ , $f''(0) = ?$

$$f'(x) = \frac{x\cos x - \sin x}{x^2}$$

$$f'(x) = \frac{x - \tan x}{x^2 \sec x}$$

$$f'(x) = \begin{cases} \dfrac{x\cos x - \sin x}{x^2} & , \ x \neq 0 \\ 0 & , \ x = 0 \end{cases}$$

$$\lim_{x\to 0} f'(0) = \lim_{x\to 0} \left(\frac{x - \tan x}{x \times x^2 . \cos x}\right)^x$$

$f'(x) = 0$

LHP

$$f''(0^+) = \lim_{h\to 0} \frac{f'(0+h) - f(0)}{h}$$

$$f''(0^+) = \lim_{h\to 0} \frac{\frac{h\cos h - \sin h}{h^2} - 0}{h}$$

$$f''(0^+) = \lim_{h\to 0} \frac{h - anh}{h^3 \cos h} = -\frac{1}{3}$$

$$f''(0) = \lim_{h\to 0} \frac{\frac{-\cos h + \sin h}{h^2} - 0}{-h}$$

$$f''(0) = \lim_{h\to 0} \frac{h - \tan h}{h^3 \cos h} = \frac{-1}{3}$$

$$f''(0) = f''(0^-) = \frac{-1}{3}$$

**Sol.5** $z = \ell n \left(\tan \frac{x}{2}\right)$

$$\frac{d^2y}{dx^2} + \cot x \frac{dy}{dx} + 4y \operatorname{cosec}^2 x = 0$$

$$z = \ell n\left(\tan\frac{x}{2}\right), \frac{d^2y}{dx^2} + \cot x \frac{dy}{dx} + 4y \operatorname{cosec}^2 x = 0$$

$$\frac{d^2y}{dz^2} + 4y = 0$$

$$\frac{dy}{dx} = \frac{dy}{dz} \cdot \frac{dz}{dx} \Rightarrow \operatorname{cosec} x \frac{dy}{dz} \ ... (1)$$

$$\frac{d^2y}{dx^2} = -\operatorname{cosec} x \cot x \frac{dy}{dx} + \operatorname{cosec} x \frac{d^2y}{dz^2} \cdot \frac{dz}{dx}$$

$$\frac{d^2y}{dx^2} = -\text{cosec } x \cot x \ \frac{dy}{dz} + \text{cosec}^2 x \ . \ \frac{d^2y}{dz^2} \quad ...(2)$$

$$\frac{d^2y}{dx^2} + \cot x \ . \ \frac{dy}{dx} + 4y\ \text{cosec}^2 x = 0$$

$$-\text{cosec } x \cot x \ \frac{dy}{dz} + \text{cosec}^2 x \ . \ \frac{d^2y}{dz^2} +$$

$$\cot x \ \text{cosec} x \ \frac{dy}{dz} + 4y\ \text{cosec}^2 x = 0$$

$$\frac{d^2y}{dz^2} + 4y = 0$$

**Sol.6** $\cos x + \cos 3x + \cos 5x + ... + \cos(2n-1)x = \dfrac{\sin 2nx}{2 \sin x}$

$\cos x + \cos 3x + ... + \cos(2n-1)x$

$[2 \sin x \cos x + 2 \sin x \cos 3x + 2 \sin x \cos 5x$

$+ ... 2 \sin x \cos (2n-1)x]\ \dfrac{1}{2 \sin x}$

$$\frac{[\sin 2x + \sin 4x + \sin(-2x) + \text{I}\sin 6x - \sin 4x ... \sin 2nx - \sin(2n-1)x]}{2 \sin x}$$

$\dfrac{\sin 2nx}{2 \sin x}$ = RHS

**Sol.7** Clearly f(x) is a cubic polynomial

Let $f(x) = ax^3 + bx^2 + cx + d$

$f'(x) = 3ax^2 + 2bx + c$

$f''(x) = 6ax + 2b$

Now

$f(2x) = f'(x) \ . \ f''(x)$

comparing coefficients.

$f(x) = \dfrac{4x^3}{9}$

**Sol.8** **(i) (a)** $f'(0^-) = f'(0^+) = 1$

so differentiable as will as continious

**(b)** Now differentiate & check the differentiable for $f'(x)$.

**(ii)** $f'(0^+) = \lim\limits_{h\to 0} \dfrac{h^3(1-h).\sin 1/h - 0}{h}$

$= \lim\limits_{h\to 0} h^2 \underbrace{(1-h) \sin 1/h}_{\text{finite}} = 0$

so differentiable in [0, 1]

**Sol.9** $f\left(\dfrac{x+y}{k}\right) = \dfrac{f(x)+f(y)}{k}$

Let x = kx, y = 0

$\Rightarrow$ f(kx) = k f(x) .......(i)

$f'(x) = \lim\limits_{h\to 0} \dfrac{f(x+h)-f(x)}{h}$

$= \lim\limits_{h\to 0} \dfrac{f(kx)+f(kh)-kf(x)}{kh}$ (using (i))

$f'(x) = \lim\limits_{h\to 0} \dfrac{f(kh)}{kh}$ (Integrate to get f(x))

(Now check for different power of h)

**Sol.11**

Differentiate w.r.t. each column. Separately.

**Sol.12**

$$f'(x) = \begin{vmatrix} 1 & 1 & 1 \\ l+1 & m+x & n+x \\ p+x & q+x & r+x \end{vmatrix} + \begin{vmatrix} a+x & b+x & c+x \\ 1 & 1 & 1 \\ p+x & q+x & r+x \end{vmatrix}$$

$$+ \begin{vmatrix} a+x & b+x & c+x \\ l+x & m+x & n+x \\ 1 & 1 & 1 \end{vmatrix}$$

Apply in each deterniment that $c_1 \to c_1 - c_3$

& $c_2 \to c_2 - c_3 \Rightarrow f'(x)$ = constant

$\Rightarrow f''(x) = 0$

**Sol.13** $\lim\limits_{x\to 0} \left[\dfrac{1}{x \sin^{-1} x} - \left(\dfrac{1-x^2}{x^2}\right)\right]$

$$\lim_{x\to 0} \frac{x^2 - x\sin^{-1}x + x^3 \sin^{-1} x}{x^3 \sin^{-1} x}$$

$$\lim_{x\to 0} \frac{x^2 - x\sin^{-1}x + x^3 \sin^{-1} x}{x^4}$$

$$\lim_{x\to 0} \frac{x - \sin^{-1}x + x^2 \sin^{-1} x}{x^3}$$

$$\lim_{x\to 0}\frac{1-\frac{1}{\sqrt{1-x^2}}+\frac{x^2}{\sqrt{1-x^2}}+2x\sin^{-1}x}{3x^2}$$

$$\lim_{x\to 0}\frac{x-\sin^{-1}x+x^2\sin^{-1}x}{x^3}$$

$$\lim_{x\to 0}\frac{x-x-\frac{x^3}{3!}+x^{-3}}{x^3} = 1-\frac{1}{6}=\frac{5}{6}$$

**Sol.14** $$\lim_{x\to 0}\frac{x\cos x-\ell n(1+x)}{x^2}$$

$$\lim_{x\to 0}\frac{-x\sin x-\frac{1}{(1+x)}}{+2x}$$

$$\lim_{x\to 0}\frac{-x\cos x-\sin x+\frac{1}{(1+x)^2}}{+2}$$

$$0-0+\frac{1}{+2}=\frac{+1}{2}$$

**Sol.15** **Method (i) :** use x = a + h
& then binomial expension

**Method (ii) :** $\frac{0}{0}$ form so use L'Hospital

**Sol.16** $$\lim_{x\to 0}\frac{1+\sin x-\cos x+\ell n(1-x)}{x\tan^2 x}$$

$$\lim_{x\to 0}\frac{1+\sin x-\cos x+\ln(1-x)}{x^3}$$

$$\lim_{x\to 0}\frac{\cos x+\sin x-\frac{1}{1-x}}{3x^2}$$

$$\lim_{x\to 0}\frac{-\sin x+\cos x-\frac{1}{(1-x)^2}}{6x}$$

$$\lim_{x\to 0}\frac{-\cos x-\sin x\frac{-1}{(1-1)^3}}{6}=\frac{-3}{6}=\frac{-1}{2}$$

**Sol.17** $$\lim_{x\to 0}\frac{(a+b\cos x)x-c\sin x}{x^5}=1$$

$$\lim_{x\to 0}\frac{a+b\cos x-bx\sin x-c\cos x}{5x^4}=1$$

$$\frac{a+b-c}{0}\quad\Rightarrow\quad a+b=c$$

$$\lim_{x\to 0}\frac{a+b\cos x-bx\sin x-(a+b)\cos x}{5x^4}=1$$

$$\lim_{x\to 0}\frac{a(1-\cos x)-bx\sin x}{5x^4}=1$$

$$\lim_{x\to 0}\frac{a\left(\frac{1-\cos x}{x^2}\right)-b\left(\frac{\sin x}{x}\right)}{5x^2}=1$$

$$\frac{\frac{a}{2}-b}{0}\quad\Rightarrow\quad a=2b$$

$$\lim_{x\to 0}\frac{2b-2b\cos x-bx\sin x}{5x^4}=1$$

$$\frac{2b-2b\left(1-\frac{x^2}{2!}+\frac{x^4}{4!}\right)-bx\left(x-\frac{x^3}{3!}\right)=1}{5x^4}$$

$$\frac{\frac{2bx^2}{2!}-\frac{2bx^4}{4!}-bx^2+\frac{bx^4}{3!}}{5x^4}=1$$

$$\left(\frac{-2b}{4!}+\frac{b}{3!}\right)=5$$

$$\frac{-2b}{4\times 6}+\frac{b}{6}=5$$

$$\frac{-2b}{4}+b=30$$

2b = 120
a = 2b = 2 × 60 = 120
a + b = c $\Rightarrow$ c = 120 + 60 = 180

**Sol.18** Let sin x = t

$$L=\lim_{t\to 1}\frac{t-t^t}{1-t+\ell n\,t}\qquad \text{(using L' Hospital)}$$

$$L=\lim_{t\to 1}\frac{1-t^t(1+\ell n\,t)}{-1+1/t}\qquad \text{(again using L' Hospital)}$$

$$= \lim_{t\to 1} -\frac{(t^t(1/t)+(1+\ell n\, t)\, t^t}{-1/t^2} = 2$$

**Sol.19** $\lim\limits_{x\to 0} \dfrac{3x\ln\left(\dfrac{\sin x}{x}\right)^2 + x^3}{(x-\sin x)(1-\cos x)}$

$$\lim_{x\to 0} \frac{3x\ln\left(\dfrac{\sin x}{x}\right)^2 + x^3}{\dfrac{(x-\sin x)}{x^3}\times\left(\dfrac{1-\cos x}{x^2}\right)\times x^5}$$

$$\lim_{x\to 0} \frac{3x\ln\left(\dfrac{\sin x}{x}\right)^2 + x^3}{\dfrac{1}{6}\times\dfrac{1}{2}\times x^5}$$

$$\lim_{x\to 0} \frac{3\ln\left(\dfrac{\sin x}{x}\right)^2 + x^2}{x^4}$$

**Sol.20** As f(x) is continious

$$f(0^+) = \lim_{x\to 0} \frac{1}{x} - \frac{2}{e^{2x}-1}$$

$$= \lim_{x\to 0} \frac{e^{2x}-1-2x}{x(e^{2x}-1)} \qquad (\%\ \text{form})$$

$f(0^+) = 1 = f(0)$ ($\because$ function is continous)

$$f'(0^+) = \lim_{h\to 0} \frac{f(0+h)-f(0)}{h} = -\frac{1}{3} = f'(0^-)$$

**Sol.21** $L = \lim\limits_{x\to 0} \dfrac{a\sin x - bx + cx^2 + x^3}{2x^2.\ell n(1+x) - 2x^3 + x^4}$ $\left(\dfrac{0}{0}\text{ form}\right)$

Using L' Hospital

$$L = \lim_{x\to 0} \frac{a\cos x - b + 2cx + 3x^2}{4x\ \ell n(1+x) + \dfrac{2x^2}{1+x} - 6x^2 + 4x^3}$$

for the existance of limit a – b = 0 .......(i)

$$\text{so ;}\qquad L = \lim_{x\to 0} \frac{a(\cos x - b) + 2cx + 3x^2}{4x\ \ell n(1+x) + \dfrac{2x^2}{1+x} - 6x^2 + 4x^3}$$

again using L' Hospital

$$L = \lim_{x\to 0} \frac{-a\sin x + 2c + 6x}{4\ell n(1+x) + \dfrac{4x}{1+x} + \dfrac{(4x+2x^2)}{(1+x)^2} - 12x + 12x^2}$$

for the existance of limit

c = 0

$$\&\ L = \lim_{x\to 0} -\frac{a\sin x + 6x}{4\ell n(1+x) + \dfrac{4x}{1+x} + \dfrac{(4x+2x^2)}{(1+x)^2} - 12x + 12x^2}$$

(0/0 form)

using L' Hospital & form the existance of Limit , a = 6

so b = 6 & $L = \dfrac{3}{40}$

**Sol.22** $\lim\limits_{x\to 0} \dfrac{x^{6000} - (\sin x)^{6000}}{x^2.(\sin x)^{6000}}$ $\left(\dfrac{0}{0}\text{ form}\right)$

$$= \lim_{x\to 0} \frac{x^{6000} - (\sin x)^{6000}}{x^{6002}}$$

$$= \lim_{x\to 0} \frac{1-\left(\dfrac{\sin x}{x}\right)^{6000}}{x^2} \qquad \text{(Apply L' Hospital)}$$

$$= \lim_{x\to 0} \frac{-6000\left(\dfrac{\sin x}{x}\right)^{5999}\left(\dfrac{x\cos x - \sin x}{x^2}\right)}{2x}$$

$$= \lim_{x\to 0} -6000\left(\frac{\sin x}{x}\right)^{5999} \frac{\cos x}{2}\left[\frac{x-\tan x}{x^3}\right]$$

$= 1000$

**Sol.23** $\lim\limits_{x\to 0} \dfrac{1-\cos x.\cos 2x.\cos 3x....\cos nx}{x^2}$ $\left(\dfrac{0}{0}\text{ form}\right)$

Apply L' Hospital

$$\lim_{x\to 0} \frac{\sin x.\cos 2x.\cos 3x....\cos nx + 2\sin 2x.\cos x...\cos nx \times ... \times n\cos x.\cos 2x - 2nx}{2x}$$

$$= \frac{1}{2}\lim_{x\to 0}\left[\left(\frac{\sin x}{\pi}\cos 2x...\cos x\right)\times\frac{2^2\sin 2x}{2x}\cos x...\cos x + ... + \frac{n^2}{nx}\left(\frac{1^2+2^2+3^2+...+n^2}{2}\right)\right]$$

# EXERCISE – V HINTS & SOLUTIONS

**Sol.1** $f(x) = \dfrac{x^2 - x}{x(x+2)}$

$y = \dfrac{x(x-1)}{x(x+2)}$ Domains $x \in R - \{0, -2\}$

$y = \dfrac{x-1}{x+2}$ $\quad xy + 2y = x - 1$

at $x = 0$, $y = -\dfrac{1}{2}$ $\quad x = \dfrac{-1-2y}{y-1}$

$y \neq 1$

$y \in R - \left\{-\dfrac{1}{2}, 1\right\}$

f(x) is linear / linear

It wan't repeat any value so one-one

$f^{-1}(x) = \dfrac{2x+1}{1-x}$

$\dfrac{d}{dx}(f^{-1}(x)) = \dfrac{(1-x)(2)+(2x+1)}{(1-x)^2} = \dfrac{3}{(1-x)^2}$

**Sol.2** **(a)** $x^2 + y^2 = 1$

$2x + 2yy' = 0$

$x + yy' = 0$

$1 + yy'' + (y')^2 = 0$

**(b)** $P(x) = a_0 + a_1x + a_2x^2 + ....... + a_n x^n$

$P'(1) = a_1 + 2a_2 + 3a_3 + ....... + na_n$

$|P(x)| \leq |e^{x-1} - 1|$

$|P'(x)| \leq |(1)\, e^{x-1}|$

$|P'(1)| \leq 1$

$|a_1 + 2a_2 + 3a_3 + ........ + na_n| \leq 1$

**Sol.3** **(a)** $\ell n\,(x + y) = 2xy$

$\dfrac{1}{(x+y)}(1 + y') = 2y + 2xy'$

$\dfrac{(1+y')}{y} = 2y$ $\quad \ell n\,(0 + y) = 0$

$\Rightarrow y = 1$

$1 + y' = 2y^2$

$y' = 2y^2 - 1$

$y' = 2 - 1 = 1$

**(b)** $f(0^+) = \lim\limits_{h\to 0} \dfrac{f(0+h) - f(0)}{h}$

$= \lim\limits_{h\to 0} \dfrac{\dfrac{e^{ah/2}-1}{h} - \dfrac{1}{2}}{h} = \lim\limits_{h\to 0} \dfrac{2e^{ah/2} - 2 - h}{2h^2}$

$= \lim\limits_{h\to 0} \dfrac{2\left(\dfrac{a}{2}\right)e^{ah/2} - 1}{4h} = \lim\limits_{h\to 0} \dfrac{\left(\dfrac{a^2}{2}\right)e^{ah/2}}{4} = \dfrac{a^2}{8}$

$f'(0^-) = \dfrac{b}{\sqrt{1-\left(\dfrac{x+c}{2}\right)^2}} \cdot \dfrac{1}{2} = \dfrac{b}{2\sqrt{-\dfrac{c^2}{4}}}$

It will be cont. also

$f(0) = f(0^+)$

$\dfrac{1}{2} = \lim\limits_{h\to 0^+} \left(\dfrac{a}{2}\right) \dfrac{e^{ax/2} - 1}{\left(\dfrac{ax}{2}\right)}$

$\dfrac{a}{2} = \dfrac{1}{2} \Rightarrow a = 1$

$f'(0^+) = f'(0^-)$

$\dfrac{1}{8} = \dfrac{b}{2\sqrt{1-\dfrac{c^2}{4}}} \Rightarrow 4b = \sqrt{1-\dfrac{c^2}{4}}$

$\Rightarrow 16b^2 = 1 - \dfrac{c^2}{4} \Rightarrow 6 + b^2 = 4 - c^2$

**Sol.4** **(a)** $y = y(x)$ $\quad x \cos y + y \cos = \pi$

$0 + y = \pi$ ; $y = \pi$ when $x = 0$

$y'(0) = \cos y = \cos x = -1$

$\cos y - x \sin y \,.\, y' + y' \cos x - y \sin x = 0$

$\cos y + y' (\cos x - x \sin y) - y \sin x = 0$

$- \sin y . y' + y''(\cos x - x \sin y) + y' (-\sin x - \sin y - x \cos y) - y' \sin x - y \cos x = 0$

$\Rightarrow - \sin y . y' + y'' - \sin yy' - y = 0$

$y''(0) - 2 \sin x \,.\, y' - \pi = 0$

$y''(0) = \pi$

**(b)** Let the polynomial $P(x) = ax^2 + bx + c$

$P(0) = 0 \Rightarrow c = 0$

$P(1) = 1 \Rightarrow (a + b) = 1$ so that

$P'(x) = 2(1 - b)x + b > 0 \;\forall\; x \in [0, 1]$

$b \in (0, 2)$

$S = \{(1 - a)x^2 + ax, a \in (0, 2)\}$

**(c)** as $n \to \infty$

$f(0) = 0$ & if function is continuos

$\Rightarrow f'(0) = 0$

**(d)** $f(x-y) = f(x).g(y) - f(y)\,g(x)$ .....(1)

put x = y

$f(0) = 0$

put y = 0 in (1) we get

$g(0) = 1$

Now $f'(0^+) = \lim\limits_{h\to 0} \dfrac{f(0+h)-f(0)}{h}$

$= \lim\limits_{h\to 0} \dfrac{f(0)\,g(-h)-g(0)\,f(-h)-f(0)}{h}$

$= \lim\limits_{h\to 0} \dfrac{f(-h)}{-h}$ $\quad (\because f(0)=0)$

$= \lim\limits_{h\to 0} \dfrac{f(0-h)-f(0)}{-h}$

$f'(a^+) = f'(0^-)$

Hence f(x) is diff. at x = 0

$g(x-y) = g(x).g(y) + f(x).f(y)$

put y = x

$g(0) = g^2(x) + f^2(x)$

$1 = g^2(x) + f^2(x)$

$g^2(x) = 1 - f^2(x)$

$2g'(0)\,g(0) = -2f(0)\,f'(0)$

$g'(0) = 0$

**Sol.5** $\lim\limits_{x\to 0} \left((\sin x)^{1/x} + \left(\dfrac{1}{x}\right)^{\sin x}\right)$

$\downarrow$

$0^\infty$ is not an indeterminent term

$L = \lim\limits_{x\to 0} \left(\dfrac{1}{x}\right)^{\sin x}$

$\ell n\, L = \lim\limits_{x\to 0} \sin x\, \ell n \left(\dfrac{1}{x}\right)$

$= -\lim\limits_{x\to 0} \sin x\, \ell n\, x$

$\ell n\, L = 0$

$L = 1$

**Sol.6** $\dfrac{d^2x}{dy^2} = ?$

$\dfrac{dx}{dy} = \left(\dfrac{dy}{dx}\right)^{-1}$

$\dfrac{d^2x}{dy^2} = \dfrac{d}{dy}\left(\dfrac{dy}{dx}\right)^{-1} = \dfrac{d}{dx}\left(\dfrac{dy}{dx}\right)^{-1} . \dfrac{dx}{dy}$

$= -\left(\dfrac{dy}{dx}\right)^{-2} . \dfrac{d^2y}{dx^2} . \left(\dfrac{dy}{dx}\right)^{-1} = -\dfrac{d^2y}{dx^2} . \left(\dfrac{dy}{dx}\right)^{-3}$

**Sol.7** **(a)** $g(x+1) = \ell n\, f(x+1) = \ell n\,(x\, f(x))$

$= \ell n\, x + \ell n\, f(x)$

$= \log x + g(x)$

$g(x+1) - g(x) = \log x$

$g''(x+1) - g''(x) = -\dfrac{1}{x^2}$

$g''\left(1+\dfrac{1}{2}\right) - g''\left(\dfrac{1}{2}\right) = -4$

$g''\left(2+\dfrac{1}{2}\right) - g''\left(1+\dfrac{1}{2}\right) = -\dfrac{4}{9}$

$\vdots$

$g''\left(N+\dfrac{1}{2}\right) - g''\left(N-\dfrac{1}{2}\right) = -\dfrac{4}{(2N-1)^2}$

$g''\left(N+\dfrac{1}{2}\right) - g''\left(\dfrac{1}{2}\right) = -4\left\{1+\dfrac{1}{9}+....+\dfrac{1}{(2N-1)^2}\right\}$

**(b)** $f(x) = g(x)\sin x$

$f'(x) = g(x)\cos x + \sin x\,.\,g'(x)$

$f'(0) = g(0)$

$f''(x) = 2\,g'(x)\cos x - g(x)\sin x + \sin x\, g''(x)$

$f''(0) = 2\,g'(0) = 0$

$\lim\limits_{x\to 0} [g(x)\cot x - g(0)\operatorname{cosec} x]$

$= \lim\limits_{x\to 0} \dfrac{g(x)\cos x - g(0)}{\sin x}$

$= \lim\limits_{x\to 0} \dfrac{g'(x)\cos x - g(x)\sin x}{\cos x}$

$= g'(0) = 0 = f''(0)$

**Sol.8**

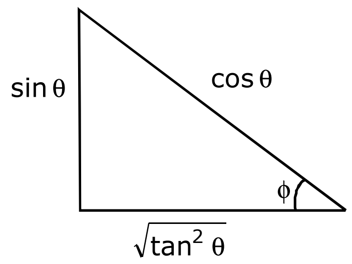
$f(x) = x^3 + e^{x/2}$ $\qquad y = x^3 + e^{x/2}$

$g'(y) = \dfrac{1}{f'(x)}$ $\qquad y = 1,\ x = 0$

$g'(y) = \dfrac{1}{3x^2 + \dfrac{e^{x/2}}{2}}$

$$g'(y)\big|_{y=1} = \left.\frac{1}{3x^2 + \frac{e^{x/2}}{2}}\right|_{x=0} = 2$$

**Sol.9**




So $f(\theta) = \sin(\sin^{-1}(\tan\theta)) = \tan\theta$

so $\frac{d}{d(\tan\theta)} f(\theta) = 1$

**Sol.10** $\frac{dy}{dx} + g'(x)\, y = g(x)\,.\, g'(x)$

I.F. = $e^{g(x)}$

solution $y\,.e^{g(x)} = \int g(x).g'(x).e^{g(x)}dx$

$\Rightarrow \quad ye^{g(x)} = (g(x) - 1)e^{g(x)} + c$

$x = 0 \qquad c = 1$

$\Rightarrow \quad y = g(x) - 1 + e^{-g(x)}$

at $x = 2 \qquad \Rightarrow y(2) = 0$

## Answer Ex–I SINGLE CORRECT (OBJECTIVE QUESTIONS)

| | | | | | | |
|---|---|---|---|---|---|---|
| **1.** C | **2.** A | **3.** B | **4.** C | **5.** B | **6.** C | **7.** D |
| **8.** C | **9.** B | **10.** D | **11.** D | **12.** B | **13.** D | **14.** C |
| **15.** B | **16.** C | **17.** B | **18.** B | **19.** C | **20.** C | **21.** C |
| **22.** C | **23.** B | **24.** C | **25.** D | **26.** A | **27.** C | **28.** D |
| **29.** D | **30.** D | **31.** B | **32.** B | **33.** A | **34.** C | |

## Answer Ex–II MULTIPLE CORRECT (OBJECTIVE QUESTIONS)

| | | | | | | |
|---|---|---|---|---|---|---|
| **1.** AB | **2.** BCD | **3.** ABD | **4.** ABCD | **5.** BC | **6.** AC | **7.** CD |
| **8.** BCD | **9.** ABCD | **10.** BC | **11.** ABC | **12.** AB | | |

## Answer Ex–III SUBJECTIVE QUESTIONS

**1. (i)** $f'(x) = 2x \cos x^2$ **(ii)** $f'(x) = 2\, e^{2x+3}$

**2. (i)** $\frac{2}{3x^{\frac{1}{3}}} + \frac{5}{x^2} + 7 \sec^2 x$ **(ii)** $e^x\, x\, (2\, ln\, x + 1 + x\, (ln\, x)$ **(iii)** $\sec x$

**(iv)** $\frac{x^2}{(x \sin x + \cos x)^2}$ **(v)** $\frac{1}{2} \sec^2 \frac{x}{2}$

**3.** (2, 3] **4. (i)** $-\frac{ax + hy + g}{hx + by + f}$ **(ii)** $\frac{2x - y - e^{-y} - e^x y}{x - xe^{-y} + e^x}$

**5. (i)** $(ln\, x)^{\cos x}\left(\frac{\cos x}{x \ln x} - \sin x \ln(\ln x)\right)$ **(ii)** $x^x (1 + \ln x) - \ln 2 \,.\, 2^{\sin x} \,.\, \cos x$

**(iii)** $(x \ln x)^{\ln \ln x}\ \frac{1}{x}\left(1 + \ln(\ln x)\left(1 + \frac{2}{\ln x}\right)\right)$ **9.** $2^n$ **10.** a = 6, b = 6, c = 0; $\frac{3}{40}$

**11.** $\mathrm{cosec}^2 x - (1/x^2)$ **14.** $Dy = (\cos x)^{\ln x}\left[\frac{\ln(\cos x)}{x} - \tan x \ln x\right] + (\ln x)^x \left[\frac{1}{\ln x} + \ln(\ln x)\right]$

**15. (a)** $\left(-\frac{1}{2}, \frac{1}{2}\right)$, $(-\infty, \infty)$ **(b)** $f(x) = \frac{2x}{\sqrt{1 - 4x^2}}$ **(c)** $\frac{16\sqrt{3}}{9}$ **16.** 100

**17.** $\dfrac{1+\sqrt{1+x^4}}{x^6}$ **18.** $\dfrac{32}{16+\pi^2}-\dfrac{8}{\ln 2}$ **22.** $\dfrac{1}{2}$ or $-\dfrac{1}{2}$

**23.** $\dfrac{1-2x}{2\sqrt{1-x^2}}$ **24. (a)** 1/6 ; **(b)** 16 **25.** $-\dfrac{3}{2}$

**28.** $f(x) = \begin{cases} -\dfrac{2}{3}\left[\dfrac{1}{6}+\ln\dfrac{3}{2}\right]x & \text{if} \quad x \le 0 \\ \left(\dfrac{1+x}{2+x}\right)^{1/x} & \text{if} \quad x > 0 \end{cases}$ **30.** $\dfrac{1}{1+(x+n)^2}-\dfrac{1}{1+x^2}$

## Answer Ex–IV ADVANCED SUBJECTIVE QUESTIONS

**2.** 6 **3.** k = 1, –1 or 0 **4.** $f'(x) = \begin{cases} \dfrac{x\cos x - \sin x}{x^2} & \text{if} \quad x \ne 0 \\ 0 & \text{if} \quad x = 0 \end{cases}$; $f''(0) = -\dfrac{1}{3}$

**7.** $\dfrac{4x^3}{9}$ **11.** 3 **13.** $\dfrac{5}{6}$ **14.** $\dfrac{1}{2}$

**15.** a = 1 **16.** $-\dfrac{1}{2}$ **17.** a = 120; b = 60; c = 180 **18.** 2

**19.** –2/5 **20.** f(0) = 1; differentiable at x = 0, $f'(0^+) = -(1/3)$; $f'(0^-) = -(1/3)$

**21.** a = 6, b = 6, c = 0; $\dfrac{3}{40}$ **22.** 1000 **23.** n = 11

## Answer Ex–V JEE PROBLEMS

**1.** Domain of f(x) = R – {–2, 0} ; Range of f(x) = R – {–1/2, 1} ; $\dfrac{d}{dx}\,[f^{-1}(x)] = \dfrac{3}{(1-x)^2}$

Domain of $f^{-1}(x)$ = R – {– 1/2, 1} **2. (a)** B **3. (a)** A ; **(b)** a = 1

**4. (a)** C ; **(b)** B ; **(c)** B, **(d)** $g'(0) = 0$ **5.** C **6.** D **7. (a)** A, **(b)** A **8.** 2